Mon. Not. R. Astron. Soc. **295,** 109–118 (1998)

On the physical model of dust around Wolf–Rayet stars

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Accepted 1997 October 13. Received 1997 September 4; in original form 1996 December 20

ABSTRACT

The observational infrared spectra of a number of Wolf–Rayet stars of WC8–9 spectral classes are shown to be quite satisfactorily explained by making use of the detailed theoretical model of a dust shell made up of spherical amorphous carbon grains, the dynamics, growth–destruction, thermal and electrical charge balance of which are taken into account. The dust grains acquire mainly positive electrical charge, move with suprathermal drift velocities and may grow up to 100–200 Å as a result of implantation of impinging carbon ions. For most of the stars the fraction of condensed carbon does not exceed 1 per cent. While the nature of the grain nucleation remains unknown, the condensation distances and the grain seed production can be estimated by fitting the observational spectra with theoretical ones.

Key words: methods: numerical – stars: carbon – circumstellar matter – stars: Wolf–Rayet – dust, extinction – infrared: stars.

1 INTRODUCTION

Already, early infrared studies by Allen, Harvey & Swings (1972), Gehrz & Hackwell (1974) and Cohen (1975) have clearly shown that the Wolf–Rayet (WR) stars, of the latest WC subtypes (WC 8–10) only, are surrounded by circumstellar dust shells. This conclusion was later strengthened by discovering new late-type WC stars (Williams, van der Hucht & Thé 1987, hereafter WHT). In addition, it has been found that the stars with dust may be classified in two groups: the first one contains the objects with persistent infrared radiation and, consequently, dust production whereas the second one includes the so-called 'episodic' dust-makers with highly variable infrared radiation (WHT; Williams et al. 1992). The present status of our knowledge about the 'dusty' WR stars may be found in a review by Williams (1995). What about the nature of dust around WR stars? The first attempts to explain the infrared energy distributions of WR stars by Allen et al. (1972), Gehrz & Hackwell (1974) and Cohen, Barlow & Kuhi (1975) have been based on the model of graphite grains. Later, WHT performed comprehensive modelling of a number of WC stars and found amorphous carbon to be more suitable for the explanation of the continuous energy distributions of these stars. They have estimated that grain growth is impossible in the frames of the traditional dust condensation scheme, i.e. under the conditions of thermodynamic quasi-equilibrium. It should be noted, however, that WHT's models were quite simplistic and did not take into account the details of dust formation physics. It is not clear so far where and how the grain nucleation occurs. Sedlmayr & Gass (1991) have estimated that the dust formation around WC stars may take place in the neutral carbon zones extended up to $1000r_*$. However, van der Hucht, Cassinelli & Williams (1986) have found that a WC star shell electronic temperature T_e at distances $400–1000r_*$ should be around 10^4 K. Carbon therefore should be almost entirely ionized under such conditions.

Exploring the possibility of condensation of neutral carbon, Cherchneff & Tielens (1995) have proposed a two-component WC wind model: a dense neutral equatorial disc, which may be a suitable place for the creation of dust nuclei, and a spherical, low-density, ionized outflowing shell. Perhaps the dust nucleation in the WC star shells resembles that in novae (Bode 1995)? Zubko, Marchenko & Nugis (1992) have shown that, provided the condensation nuclei are created somehow, their subsequent growth may still be possible by means of the *implantation* of impinging carbon ions even in a highly-ionized standard WC star atmosphere. The last process becomes important because of the quite high grain drift velocities even in spite of the prevailing positive electrical charge of grains. Zubko (1992) has proposed a physical theory of the dust growth and dynamics on the base of the kinetic equation approach. Assuming that the geometrical thickness of the layer of

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principal grain nucleation in a WR shell is considerably less than its distance from the star, Zubko (1992) has applied his theory to model the infrared spectra of a few stars from WHT's list and shown that the core–mantle grains with a graphite nucleus coated by a thin amorphous carbon mantle are the most favourable. Note, however, a few limitations of Zubko's (1992) study: he used the approximate equations for the grain dynamics, electrical charge and growth and modelled the near- and mid-infrared spectra (1–20 μm). The main purpose of the present paper is therefore to present the results of a homogeneous modelling of the energy distributions and dust shells for a number of WC stars from WHT's list using:

(i) our previous theoretical approach (Zubko 1992, 1995),
(ii) new, recently derived optical constants of both amorphous carbon (Zubko et al. 1996) and graphite (Zubko 1997),
(iii) more exact equations describing the grain physics, and
(iv) the *IRAS* (Infrared Astronomical Satellite) data for WC stars (Cohen 1995b).

In this paper we analyse the dust shells around the persistent WC stars. The models of dust around the 'episodic' dust makers will be considered in our forthcoming papers.

2 THEORETICAL MODEL

Similarly to Zubko (1992), we have made following assumptions.

(i) Stationarity and spherical symmetry of the problem, which is likely to be valid for the persistent WR stars.
(ii) Constant velocity of the atmospheric plasma outflow v and, as a consequence, the inverse square law for the number density distributions of all plasma components.
(iii) The atmospheric plasma at the distance of a probable dust location ($r \geq 500 r_*$, where r_* is stellar radius) may consist of such components, which are important for grain physics: the ions He^+, C^+, C^{++}, O^+ and O^{++} and electrons e^-. We assumed also a constancy of the ionization state distributions in the regions of dust growth, based on the results by van der Hucht et al. (1986). Strictly speaking, there should be radial gradients of ionization (Hamann 1995; Schmutz 1995). However, at present there are no late WC star wind models extended to distances $>$500-1000r_* that might be used as a basis for our calculations. We expect that the introduction of more detailed wind structure will favour grain growth, e.g. the larger the distance the lower the plasma temperatures, and the greater the number of C^+ ions compared with C^{++} ions.
(iv) The dust shell itself is optically thin in both ultraviolet (where the star emits) and infrared regions. As WHT have shown, it is a good approximation for most WC stars.
(v) Dust nucleation occurs in a geometrically thin layer located at the nucleation distance r_0.
(vi) The dynamics of dust is considered using the atmospheric plasma dynamics [see assumptions (ii) and (iii)] as a background. It may be justified by the following arguments: (a) as a rule the condensed carbon fractions are less than 1 per cent (see Section 3), (b) the velocity of the atmospheric outflow has already reached its asymptotic value, $\sim$1500–2000 km s^{-1}, at $r \gtrsim 100 r_*$ whereas the maximum dust drift velocities are about an order smaller, (c) the main driving factor of the atmosphere is not dust but purely the radiation pressure in some lines (see, e.g. Schmutz 1995).
(vii) The dust grains are spherical. In our previous study (Zubko 1992) we used the physical criterion of crystalline/amorphous carbon grain growth by Gail & Sedlmayr (1984) modified for usage in WR winds. This resulted in the core--mantle model of carbon grains: a graphite core coated by a thin amorphous carbon mantle. However, by now there is strong experimental evidence that the intense ion bombardment of a graphite sample produces various defects in the carbon lattice: vacancies, interstitial atoms and bond angle disorders that may lead to a complete amorphization of the lattice (Tielens et al. 1994 and references therein). The criteria of Gail & Sedlmayr (1984) and Zubko (1992) are probably inappropriate under these conditions and should be modified. We have therefore used in the calculations reported in present paper three alternative dust models: pure graphite, pure amorphous carbon and graphite core–amorphous carbon mantle grains. The purpose was to find the best models fitting the infrared stellar spectra. We have used the optical constants of graphite from Laor & Draine (1993) extrapolated to high temperatures by Zubko (1997). Since the atmospheres of WC stars are extremely hydrogen-poor (Nugis 1991, 1995) we used in our modelling the optical constants for a pure amorphous carbon (ACAR) sample (Zubko et al. 1996). To calculate the grain optical efficiencies, the Mie theory and its extension for the core–mantle grains (Bohren & Huffman 1983) have been applied.

We next consider the main equations of our theoretical model.

2.1 Grain dynamics

Our estimates show that the dynamics of an electrically charged dust grain is dominated by the stellar radiation pressure F_{pr} and the drag force F_{drag}. As the grain drift velocity u relaxes very quickly, its steady-state values were calculated from the appropriate force-balance equation:

$$F_{pr} = F_{drag}(u), \tag{1}$$

with the following expressions for forces (Draine & Salpeter 1979; Bohren & Huffman 1983):

$$F_{pr} = \frac{1}{c}\left(\frac{r_*}{r}\right)^2 \pi a^2 \int_0^\infty Q_{pr}(a, \lambda, T_d) B(\lambda, T_*)\, d\lambda, \tag{2}$$

$$F_{drag} = 2\pi a^2 kT \left\{ \sum_i n_i [G_1(s_i) + z_i^2 \Phi^2 \ln(\Lambda/z_i) G_2(s_i)] \right\}, \tag{3}$$

$$G_1(s) = \left(s^2 + 1 - \frac{1}{4s^2}\right) \operatorname{erf}(s) + \left(s + \frac{1}{2s}\right) \frac{\exp(-s^2)}{\sqrt{\pi}}, \tag{4}$$

$$G_2(s) = \frac{\operatorname{erf}(s)}{s^2} - \frac{2\exp(-s^2)}{s\sqrt{\pi}}, \tag{5}$$

where r is the distance from the star, a is the grain radius, $Q_{\rm pr}$ is the radiation pressure efficiency, B is the Planck function, $T_{\rm d}$ is the grain temperature, T_* is the stellar effective temperature, T is the plasma temperature (we suppose that the temperatures of electrons and ions are equal), $s_i=(m_i u^2/2kT)^{1/2}$, where n_i, m_i and $z_i e$ are the number density, mass and charge of the ith plasma component, $\Phi=eU/kT$, where U is the grain electrostatic potential, $\Lambda=(3/2ae\Phi)\sqrt{kT/\pi n_{\rm e}}$ is the Coulomb 'cut-off' factor with $\ln(\Lambda)$ typically $\sim$15–25 for the range of plasma parameters used in our present modelling, and $n_{\rm e}$ is the electron number density. The optical constants of graphite grains are strongly dependent on the grain temperature (see, e.g., Zubko 1997). This was taken into account when we calculated the radiation pressure efficiency $Q_{\rm pr}$ (equation 2) and the other optical efficiencies, $Q_{\rm abs}$ and $Q_{\rm ext}$, used in our modelling (equations 9, 10, 12, 24 and 25). The first term in equation (3) takes into account direct collisions with plasma particles whereas the second one is defined by the cumulative effect of small-angle Coulomb scattering on a charged grain. The summation in equation (3) expands over all plasma species adopted. Equations (3–5) are exact provided that (i) the grain size a is negligibly small compared with the mean free path length of the plasma particles, (ii) ion collisions with the grain are completely inelastic, and (iii) Coulomb focusing is neglected. The results of our present modelling are consistent with these assumptions. Finally, the radial motion of the grain obeys the equation

$$\frac{\mathrm{d}r}{\mathrm{d}t}=v+u. \tag{6}$$

As we treat the motion of charged grains, it is worth considering another possible contributor to the force-balance of the grain, the Lorenz force, provided that there is a magnetic field in the wind. However, it is not easy to estimate the relative significance of the magnetic force. There are no direct observational measurements of the magnetic field strengths in WR star winds. Some polarimetric studies (e.g. McLean et al. 1979 and Morel, St-Louis & Marchenko 1997 and references therein) show evidence *against* large magnetic fields, at least within the distances 10–50r_*. Maheswaran & Cassinelli (1988, 1992) have estimated the average surface magnetic fields of rapidly rotating and hot WR stars on the basis of the magnetic rotator model, and this resulted in kG-strengthened fields. However, the applicability of the magnetic rotator model to late WC stars, the objects of the present study, as well as to other WR stars at all, seems to be quite problematic at present. These topics are discussed by Schulte-Ladbeck (1995), Owocki & Gayley (1995) and Morel et al. (1997). A growing amount of data presents evidence in favour of the inhomogeneous, structured winds of WR stars (Schulte-Ladbeck 1995; Morel et al. 1997) reflecting the 'wind-photospheric connection' found earlier for O stars. Very probably there are no strong large-scale magnetic fields ($\geq$1 kG) in the WR and O stars except, possibly, for some regions emerging from active zones in the stellar photosphere (Cranmer & Owocki 1996). We thus assumed for our modelling that the Lorenz force acting on a dust grain is sufficiently small in comparison with the force of radiation pressure. At the same time, we calculated the critical magnetic field strengths, $B_{\rm cr}(r)$, at which the Lorenz force might become significant using a formula similar to formula (7) of Draine & Salpeter (1979). These should be compared with the expected model magnetic field strengths in the wind, $B(r)$. It is well-known that a rotating star generates a toroidal magnetic field in its deep outflowing wind (e.g. Weber & Davies 1967). Such a field could be responsible for the Lorenz force acting on a charged grain, because the field lines are perpendicular to the adopted radial motion of the grain. The strength of the magnetic field in the equatorial plane of the wind is (Usov & Melrose 1992)

$$B(r)\approx B_1\frac{r_*}{r}, \tag{7}$$

$$B_1=B_{\rm s}\frac{v_{\rm rot}}{v}\frac{r_*}{r_{\rm A}}, \tag{8}$$

where $B_{\rm s}$ is the surface magnetic field strength, $v_{\rm rot}$ is the surface rotation velocity and $r_{\rm A}$ is the so-called radius of the dead zone near the star. Note that equations (7) and (8) are valid for $r>r_{\rm A}$ which is the case represented by our modelling (see Section 3). Comparing $B(r)$ with $B_{\rm cr}(r)$ calculated from the dust models, we may therefore estimate the upper bounds of the parameter B_1 under which we may neglect the Lorenz force. This will put constraints on the possible values of $B_{\rm s}$ and $v_{\rm rot}$.

2.2 Grain temperature

The thermal balance of dust grains is governed by radiative processes. The role of collisions with plasma ions is negligible. The gain of energy, $\dot{E}^+$, occurs through absorption of the dilute stellar radiation whereas the energy lost, $\dot{E}^-$, is defined by thermal re-emission corresponding to the grain temperature $T_{\rm d}$ (see, e.g., Spitzer 1978):

$$\dot{E}^+=\pi a^2\left(\frac{r_*}{r}\right)^2\int_0^\infty Q_{\rm abs}(a,\lambda,T_{\rm d})B(\lambda,T_*)\,\mathrm{d}\lambda, \tag{9}$$

$$\dot{E}^-=4\pi a^2\int_0^\infty Q_{\rm abs}(a,\lambda,T_{\rm d})B(\lambda,T_{\rm d})\,\mathrm{d}\lambda \tag{10}$$

where $Q_{\rm abs}$ is the absorption efficiency of a grain. The grain equilibrium temperature may then be found from the energy-balance equation:

$$\dot{E}^+=\dot{E}^-. \tag{11}$$

2.3 Grain potential

The principal processes responsible for the grain charge are photoelectron emission and collisions with impinging particles: electrons and ions. The contribution of other effects, e.g. secondary electron emission and field emission of both electrons and ions, is negligible. The expression for the photoelectron current $I_{\rm ph}$ is (Spitzer 1978)

$$I_{\rm ph}=\pi a^2\left(\frac{r_*}{r}\right)^2\int_0^{\lambda_{\max}}Q_{\rm abs}(a,\lambda,T_{\rm d})\frac{\lambda B(\lambda,T_*)}{hc}y(\lambda)\,\mathrm{d}\lambda, \tag{12}$$

where $y(\lambda)$ is the photoelectron yield, often written in the form

$$y(\lambda)=y_0(1-\lambda/\lambda_0) \tag{13}$$

with λ_0 assumed to be some threshold wavelength, and $\lambda_{\rm max}$ is the threshold wavelength corresponding to a charged grain:

$$\frac{1}{\lambda_{\rm max}}=\frac{1}{\lambda_0}+\max\,(0,\,\Phi kT/hc). \tag{14}$$

In our modelling we used the values $y_0=0.05$ and $\lambda_0=0.1774$ μm (corresponding to energy 7 eV) for carbon grains, from Bel et al. (1989).

The current resulting from the ith impinging species I_i is (Draine & Salpeter 1979)

$$I_i=4\pi a^2\beta_i z_i n_i\sqrt{\frac{kT}{2\pi m_i}}\,G_3(s_i,\,z_i,\,\Phi), \tag{15}$$

where

$$G_3(s_i\,,\,z_i\,,\,\Phi)=\frac{{\rm e}^{-s_i^2}}{2s_i}\int_{\varepsilon_0}^{\infty}\sqrt{\varepsilon}\left(1-\frac{z_i\Phi}{\varepsilon}\right){\rm e}^{-\varepsilon}\sinh(2\sqrt{\varepsilon}s_i)\;{\rm d}\varepsilon, \tag{16}$$

$$\varepsilon_0=\max[0,\,z_i\Phi] \tag{17}$$

and β_i is the sticking probability of species i. Equation (16) takes account of the electrostatic repulsion or attraction between the charged grains and accreting ions or electrons. There is no threshold drift velocity of the grain below which the plasma species do not impinge. This is because equations (15–17) were derived assuming a Maxwellian energy distributions of the plasma species (Draine & Salpeter 1979). For a grain with any drift velocity and electrostatic potential therefore, there will be ions of sufficiently high energies to impinge on the grain. Thus, the equilibrium grain potential Φ obeys the following equation:

$$I_{\rm ph}+\sum_i I_i=0. \tag{18}$$

2.4 Grain growth–destruction

As the expected fraction of neutral carbon at the condensation distances in WR winds is likely to be less than or about 10^{-6} (WHT), we suppose that the growth of a carbon grain moving with a suprathermal velocity occurs through the implantation of carbon ions, which are much more abundant. In addition, the grain may be destroyed through (i) thermal evaporation and (ii) kinetic sputtering by impinging ions. The gain of carbon ions/atoms into the grain per unit square is (Draine & Salpeter 1979)

$$\dot{N}^+=\sum_i\beta_i n_i\sqrt{\frac{kT}{2\pi m_i}}\,G_3(s_i,\,z_i,\,\Phi), \tag{19}$$

where the summation is over all carbon ions and atoms. In our modelling the contribution of carbon ions C^{++} in $\dot{N}^+$ dominates. The loss of carbon atoms from the grain per unit square resulting from thermal evaporation is (Spitzer 1978)

$$\dot{N}^-_{\rm ev}=\frac{p_{\rm s}(T_{\rm d})}{\sqrt{2\pi m_{\rm c}kT_{\rm d}}}, \tag{20}$$

where $p_{\rm s}$ is the vapour pressure of the solid carbon at the grain temperature $T_{\rm d}$. The loss of carbon atoms from the grain per unit square resulting from kinetic sputtering is (Draine & Salpeter 1979)

$$\dot{N}^-_{\rm sp}=\sum_i\beta_i n_i\sqrt{\frac{kT}{2\pi m_i}}\,G_4(s_i,\,z_i,\,\Phi,\,T), \tag{21}$$

where G_4 is the same as G_3 in equation (16) except that the integrand should be multiplied by $Y_i[kT(\varepsilon-z_i\Phi)]$, the angle-averaged sputtering yield corresponding to the ith species. We calculated Y_i using the formulae and new sputtering parameters for carbon grains from a recent paper of Tielens et al. (1994). Then the equation for growth–destruction of the grain is

$$\frac{{\rm d}a}{{\rm d}t}=\Omega(\dot{N}^+-\dot{N}^-_{\rm ev}-\dot{N}^-_{\rm sp}), \tag{22}$$

where Ω is the volume per atom in the grain.

2.5 The dust shell luminosity

If we suppose $\dot{J}_{\rm d}$ to be the condensation nuclei production per unit time at the nucleation distance r_0, then the number density of grains $n_{\rm d}$ at any distance r may be calculated from the continuity equation:

$$n_{\rm d}=\frac{\dot{J}_{\rm d}}{4\pi r^2(v+u)}. \tag{23}$$

Taking into account the last equation, we may express the dust shell luminosity $L_{\rm d}$ and optical thickness $\tau_{\rm d}$ as (Zubko 1992)

$$L_{\rm d}(\lambda)=\dot{J}_{\rm d}\int_{r_0}^{\infty}4\pi a^2 Q_{\rm abs}(a,\,\lambda,\,T_{\rm d})\pi B(\lambda,\,T_{\rm d})\,\frac{{\rm d}r}{v+u}, \tag{24}$$

$$\tau_{\rm d}(\lambda)=\dot{J}_{\rm d}\int_{r_0}^{\infty}\pi a^2 Q_{\rm ext}(a,\,\lambda,\,T_{\rm d})\,\frac{{\rm d}r}{4\pi r^2(v+u)}, \tag{25}$$

where $Q_{\rm ext}$ is the extinction efficiency of a grain. The parameter $\dot{J}_{\rm d}$ was estimated by fitting the observational stellar spectra by theoretical ones. The theoretical spectra were in turn calculated by combining the contributions of both the 'star + wind' and the dust shell. We approximated the 'star + wind' spectra in the optical and infrared by a power law: $F_\lambda\sim\lambda^{-\alpha}$, where α is the so-called spectral index. This approximation for the continuous energy distributions was found to be excellent for both WN and WC stars without dust shells (Williams et al. 1990; Morris et al. 1993; Morris 1995). We estimated the spectral index α for each the star from the best-fitting model. The dust shell luminosity was calculated with equation (24).

3 RESULTS

We have modelled the observational spectra of 17 WC stars including two WC8 stars (WR 53, 117) and 15 WC9 stars (WR 59, 65, 69, 73, 76, 80, 95, 96, 103, 104, 106, 112, 118, 119 and 121). WHT's photometric data for these stars in both the optical and infrared have been dereddened and converted into the spectral fluxes using the same procedure as WHT. All the stars are contained in Williams' (1995) list of persistent dust-makers.

Because of a lack of reliable data we have adopted the following parameters for all models: the stellar radius $r_* = 10\ r_\odot$, the velocity of the outflowing wind $v = 2000$ km s^{-1}, the mass-loss rate $\dot{M}_* = 8 \times 10^{-5}\ M_\odot\ y^{-1}$, the chemical composition C/He = 0.3, O/He = 0.02 and no hydrogen (Nugis 1991, 1995), the effective stellar temperature $T_* = 22\,000$ K for WC9 stars and $T_* = 26\,000$ K for WC8 stars (van der Hucht et al. 1986). The plasma shell temperature T was varied in the range 6000–10 000 K to obtain a better fit; the radius of condensation centres was taken to be $a_0 = 4$ Å. The principal variable parameter was the nucleation distance r_0. Our fitting procedure consisted of the following steps.

(i) Choose appropriate T_*, T and α.
(ii) Choose r_0.
(iii) Calculate the dust shell model, i.e. the r-dependent a, u, T_d, U, B_{cr}, n_d and other parameters in the range from $r = r_0$ to $r = 10^7 r_*$ using the equations from Section 2.
(iv) Calibrate 'stellar + wind' luminosity using the optical spectral fluxes at wavelengths b, v from WHT, as the last ones are almost insensitive to the dust shell model.
(v) Vary $\dot{J}_d$ to obtain the best fit to the observational infrared spectrum.
(vi) Calculate L_d, τ_d and $n_d(r)$ from the $\dot{J}_d$ found.
(vii) Estimate the fitting error.
(viii) If a minimum was found then stop the calculations; otherwise to go to step (ii) or, if necessary, to step (i).

We have performed extensive calculations of the dust shell models for all above-mentioned WC stars. We have found that for all cases the models on the base of pure ACAR grains better explain the observed spectra than the models with graphite grains. This is especially true for the mid- and far-infrared wavelengths, $\lambda \gtrsim 10$–15 μm. Such a result is consistent with our expectations based on the appropriate laboratory data from Tielens et al. (1994). Fig. 1 shows typical behaviour of the theoretical spectra derived under various grain models using star WR 106 as an example. A similar picture is observed for other stars. We therefore report the models with amorphous carbon grains only. Some typical dust shell models for a WC9 star (WR 106) and a WC8 star (WR 53) are presented in Tables 1 and 2. The main results of our modelling are displayed in Fig. 2 and listed in Table 3.

First of all it should be noted that almost all spectra (Fig. 2) are quite satisfactorily explained in the frames

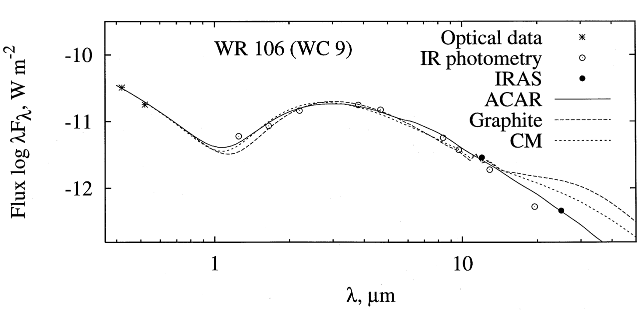

Figure 1. Theoretical energy distributions of WR 106 calculated under various dust models: amorphous carbon (ACAR), pure graphite and graphite core–amorphous carbon mantle (CM).

Table 1. The dust shell model for WR 106 (WC 9).

r	a	T_d	u	δ	U	n_d
750	4.00	1413	18.77	1.00+0	0.06	3.07–2
751	4.34	1413	19.57	1.00+0	0.09	3.06–2
757	6.21	1409	23.48	1.00+0	0.23	3.01–2
771	12.28	1400	33.17	9.48–1	0.44	2.89–2
790	21.79	1387	44.33	8.11–1	0.59	2.74–2
820	36.41	1367	57.52	5.89–1	0.71	2.52–2
865	53.77	1340	70.20	3.83–1	0.80	2.25–2
912	66.31	1313	78.18	2.68–1	0.84	2.02–2
979	78.04	1277	85.01	1.80–1	0.86	1.75–2
1014	82.49	1260	87.46	1.49–1	0.87	1.63–2
1075	88.08	1231	90.46	1.12–1	0.88	1.44–2
1160	93.29	1196	93.16	8.03–2	0.89	1.24–2
1252	96.93	1161	94.99	6.01–2	0.89	1.06–2
1416	100.74	1107	96.87	4.01–2	0.90	8.30–3
1835	104.74	1002	98.80	2.03–2	0.90	4.94–3
2678	106.92	866	99.82	1.01–2	0.90	2.32–3
5093	108.03	674	100.31	2.49–3	0.91	6.41–4
7927	108.30	568	100.41	2.03–3	0.91	2.64–4
17892	108.52	413	100.47	8.29–4	0.91	5.19–5
41281	108.61	301	100.50	5.52–4	0.91	9.75–6
287910	108.67	141	100.52	9.20–5	0.91	2.01–7
997613	108.68	83	100.52	2.76–5	0.91	1.67–8

r is the stellar radius in r_*; a is the grain radius in Å; T_d is the grain temperature in K; u is the grain drift velocity in km s^{-1}; $\delta = (\dot{N}^+ - \dot{N}^-_{ev} - \dot{N}^-_{sp})/\dot{N}^+$ characterizes the competition of the grain growth-destruction processes; U is the electrostatic potential of the grain in V; n_d is the number density of grains in cm^{-3}.

Table 2. The dust shell model for WR 63 (WC 8).

r	a	T_d	u	δ	U	n_d
800	4.00	1611	28.26	9.98–1	0.42	1.02–2
802	4.77	1610	30.92	9.96–1	0.47	1.01–2
808	7.71	1606	39.57	9.67–1	0.62	9.93–3
824	17.09	1594	59.29	7.84–1	0.83	9.46–3
865	38.36	1565	89.29	3.98–1	1.00	8.46–3
902	50.90	1539	103.08	2.44–1	1.05	7.73–3
988	66.01	1485	117.59	1.03–1	1.09	6.39–3
1063	71.83	1444	122.70	5.88–2	1.11	5.51–3
1194	76.37	1380	126.54	2.76–2	1.11	4.36–3
1319	78.21	1329	128.05	1.58–2	1.12	3.57–3
1433	79.06	1287	128.75	1.04–2	1.12	3.02–3
1641	79.81	1222	129.37	5.36–3	1.12	2.30–3
2205	80.38	1092	129.80	1.84–3	1.12	1.28–3
2888	80.53	984	129.92	9.24–4	1.12	7.43–4
4037	80.60	865	129.96	5.44–4	1.12	3.80–4
5636	80.64	761	129.98	3.19–4	1.12	1.95–4
14213	80.68	530	129.99	2.48–4	1.12	3.07–5
32216	80.70	385	129.99	1.22–4	1.12	5.97–6
72176	80.71	281	130.00	7.30–5	1.12	1.19–6
958239	80.72	104	130.00	1.42–5	1.12	6.75–9

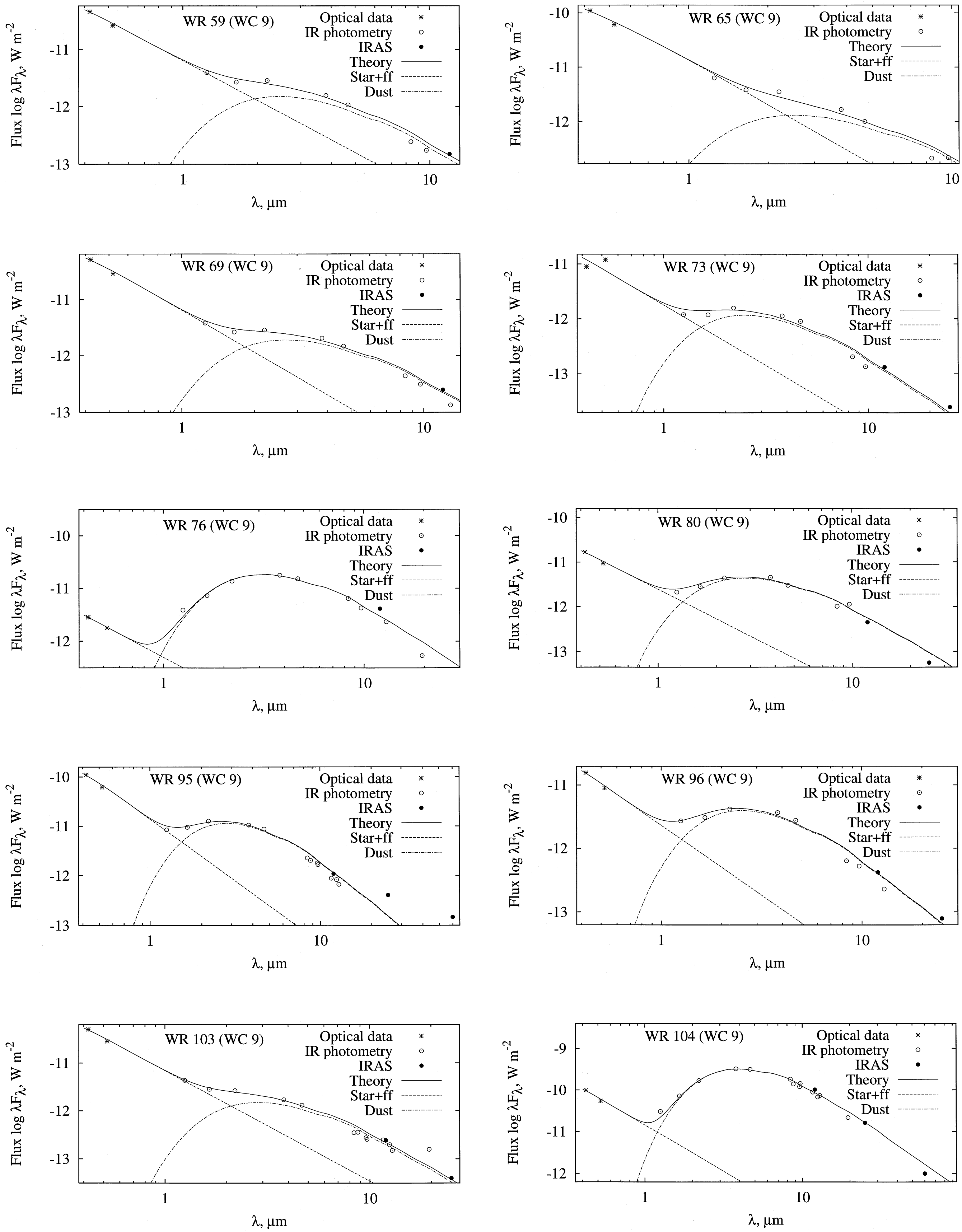

Figure 2. Observational and theoretical energy distributions of WR stars. The observational data are marked by asterisks (optical photometry), open circles (ground-based IR photometry) and filled circles (*IRAS*); the model spectra are depicted by solid lines and the contributions of 'star + wind' and dust emission by dashed and dot–dashed lines, respectively.

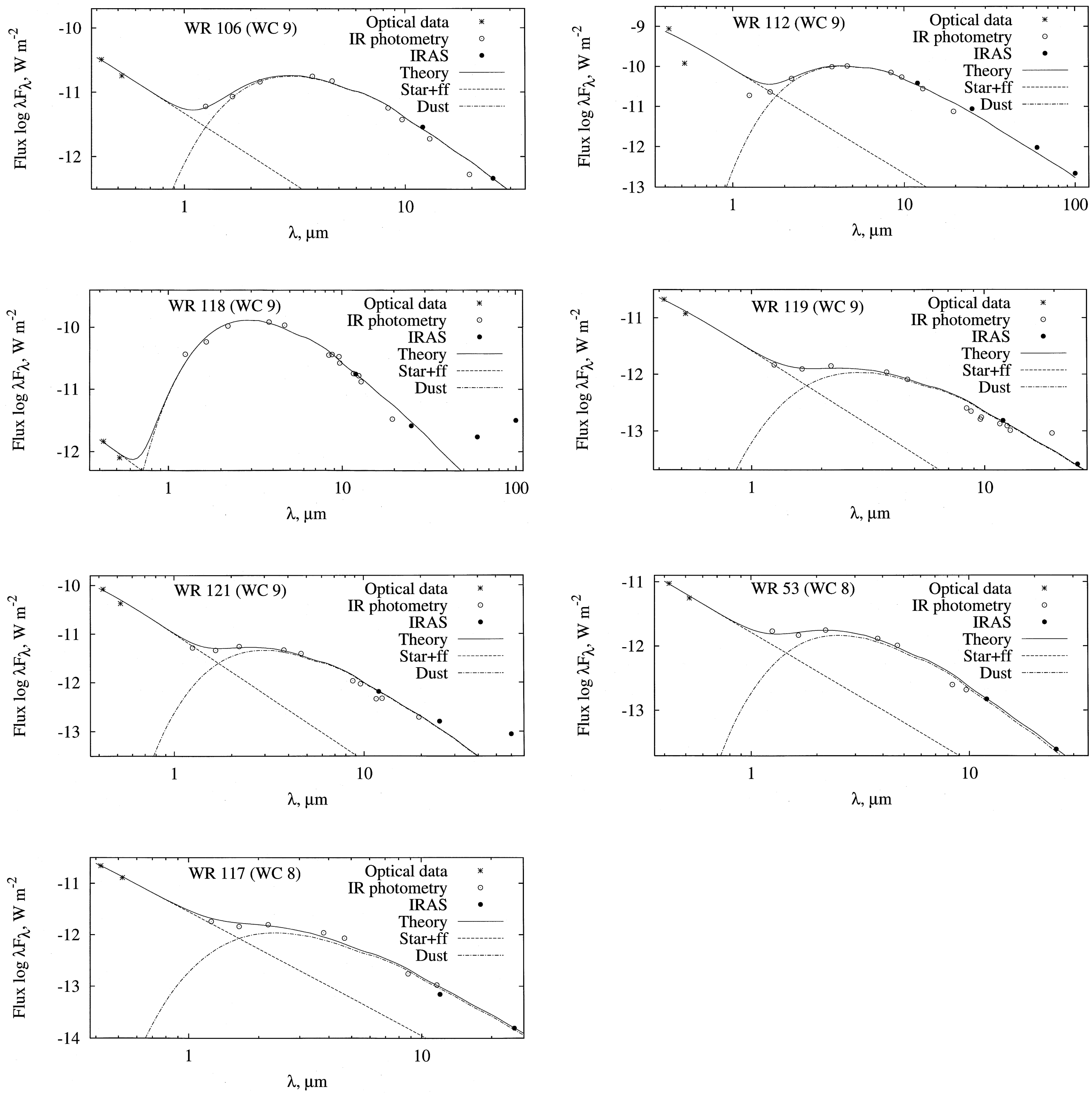

Figure 2 – *continued*

of our theoretical model. The best level of consistency is reached in the near-infrared region. Note that for some stars there are noticable deviations from the *IRAS* data in the far-infrared: for WR 95, 118 and 121 at 60 μm and, in addition, for WR 95 at 25 μm and for WR 118 at 100 μm. As in all these cases the model fluxes are lower than the corresponding *IRAS* ones we postulate some possible reasons for the deviations: additional heating of the external dust shell layers may occur after the shock passage or, alternatively, the *IRAS* fluxes may contain the contribution from other source(s) lying approximately on the same lines of sight as the stellar dust shell.

Our estimates of the spectral indices α lie in the range 3.00–3.75 (Table 3). These are on average larger than the spectral indices derived by Morris et al. (1993) for the WC stars without dust shells. While the indices derived by Morris et al. are centred at around 3.0, there are some stars from their list with $\alpha > 3.5$. Note that Morris et al. (1993) analysed three stars, WR 53, 69 and 103, having dust shells. However, they did not take the contribution of dust into account. It is therefore not surprising that their spectral indices, 1.68, 3.00 and 2.71, are lower than our respective indices, 3.0, 3.5 and 3.3.

The inner boundaries r_0 where dust nucleation is supposed to occur are mainly located at 500–1000r_*. It is

Table 3. The main parameters of the model dust shells around WC stars.

WR	Sp	T_*	T	α	r_o	T_{do}	n_{do}	a_o	a_f	u_o	u_f	U_o	U_f	$\dot{J}_d$	$\dot{M}_d$	f_d
59	WC9	22	9	3.30	500	1649	2.58–4	4	201	16	149	–0.27	0.99	8.00+32	9.61–10	0.002
65	WC9	22	6	3.75	520	1624	8.30–4	4	109	19	100	0.06	0.91	2.79+33	5.38–10	0.002
69	WC9	22	6	3.50	600	1539	2.32–3	4	109	19	100	0.06	0.91	1.04+34	1.99–9	0.006
73	WC9	22	7	3.20	500	1649	4.50–3	4	127	19	109	0.03	0.92	1.40+34	4.23–9	0.01
76	WC9	22	8	3.00	760	1407	1.70–1	4	144	19	115	–0.01	0.93	1.22+36	5.36–7	1.56
80	WC9	22	6	3.20	650	1493	1.56–2	4	109	19	100	0.06	0.91	8.21+34	1.58–8	0.05
95	WC9	22	7	3.50	750	1413	5.66–4	4	174	14	157	–0.27	1.02	3.95+33	3.09–9	0.005
96	WC9	22	8	3.20	500	1649	8.33–3	4	145	19	116	–0.01	0.93	2.59+34	1.17–8	0.03
103	WC9	22	7	3.30	650	1493	1.19–3	4	127	19	108	0.03	0.92	6.27+33	1.09–9	0.006
104	WC9	22	7	3.20	1200	1181	1.11–1	4	121	19	106	0.03	0.92	1.99+36	5.27–7	1.53
106	WC9	22	6	3.20	750	1413	3.07–2	4	109	19	100	0.06	0.91	2.15+35	4.10–8	0.12
112	WC9	22	6	3.60	1500	1085	7.08–3	4	102	19	97	0.06	0.90	1.98+35	3.14–8	0.09
118	WC9	22	8	3.00	650	1493	2.63–0	4	144	18	116	–0.01	0.93	1.38+37	6.15–6	17.9
119	WC9	22	5	3.60	700	1452	4.65–3	4	92	19	92	0.09	0.88	2.83+34	3.24–9	0.01
121	WC9	22	7	3.60	700	1452	2.18–3	4	126	19	108	0.03	0.92	1.33+34	3.99–9	0.01
53	WC8	26	10	3.00	800	1611	1.02–2	4	81	28	130	0.42	1.12	8.14+34	6.37–9	0.02
117	WC8	26	10	3.40	700	1695	3.45–3	4	81	28	130	0.42	1.12	2.11+34	1.65–9	0.005

WR numbers from the Catalogue of WR stars by van der Hucht et al. (1981); T_* is the stellar effective temperature in 10^3 K; T is the temperature of outflowing plasma in the region of dust formation in 10^3 K; α is the spectral index of the continuous energy distribution $F_\lambda \sim \lambda^{-\alpha}$ of 'star + wind' in the optical and near-IR; r_0 is the inner boundary of the dust shell in stellar radii r_*; T_{d0}, n_{d0}, $a_0(a_f)$, $u_0(u_f)$ and $U_0(U_f)$ are the temperature in K, number density in cm^{-3}, radius in Å, drift velocity in km s^{-1} and electrostatic potential in V of grains located at the initial distance r_0 (at large distances $r > 10^5 r_*$); $\dot{J}_d$ is the condensation nuclei production in s^{-1}; $\dot{M}_d$ is the mass loss due to dust in $M_\odot$ y^{-1}; f_d is the fraction of condensed carbon in per cent.

worthwhile to note that the respective models with graphite grains have values of r_0 approximately twice those for models with amorphous carbon grains.

The dust grain temperatures vary widely throughout the shell: from 1000–1700 K at the nucleation distance r_0 and down to less than 100 K at $r > 10^6 r_*$.

The initial grain radius $a_0 = 4$ Å that we have adopted seems to be somewhat artificial. However, we do not know hitherto the details of the nucleation process. We have therefore adopted some probable value close to the molecular sizes. It turned out that the final grain sizes exceed a_0 by a great amount and the models are not sensitive to the choice of a_0. The final grain radii (a_f) tend to drop from 90–200 Å for WC9 stars down to ~ 80 Å for WC8 stars. Note that these sizes are approximately 2–3 times larger than those from our previous models based on graphite grains (Zubko 1992). Analysing the dynamics of grain growth, we can see (Tables 1 and 2) that grain growth slows down rapidly when the rate of sputtering approaches that of growth. Typically this occurs at distances $< 2000 r_*$. The competition of the grain growth–destruction processes is described by the behaviour of the parameter δ (Tables 1 and 2). Starting from values close to 1 for the smallest initial grains undergoing no noticeable destruction, it slowly drops down to 0, but remains positive for the largest distances we calculated. Typically $\delta \sim 10^{-5}$ at $r \sim 10^6 r_*$. By calculating the models with smaller steps, we checked out that these non-zero values do not depend on the details of the numerical representation.

A WC star dust shell may be roughly divided into a region of intense grain growth, where the grains form, and a region of grain 'flying-away', where the grain radii, drift velocities and electrical potentials reach their asymptotic values.

The steady-state grain drift velocities vary from 10–30 km s^{-1} for the smallest grains up to 90–160 km s^{-1} for the grains at quite large distances. As a typical thermal velocity in a plasma shell is 5–10 km s^{-1}, we may conclude that the grains move with suprathermal drift velocities.

The dust grains acquire mainly positive electrical charge, while the smallest ones may be negatively charged. The principal process resulting in a positive charge is photoelectron emission. The electrical potential of the grains ranges from -0.3 to 1–1.2 V.

The condensation centre production varies widely in the range 10^{33}–10^{37} s^{-1}. It is evident that the mechanism of nucleation to be found should be consistent with these values.

The typical mass-loss rate resulting from dust is 10^{-9}–10^{-6} $M_\odot$ yr^{-1}. It results in the fractions of condensed carbon being mainly less than 1 per cent except for three stars: WR 76, 104 and 118 (1.56, 1.53 and 17.9 per cent, respectively). Therefore, our assumption (vi) (Section 2) is principally justified. Note that for the above-mentioned three stars the optical depths of dust are around 1 in the ultraviolet, where the stellar energy distribution and carbon dust opacity reach their maxima. Our models for these cases should therefore be considered an approximation. For the rest of the stars, τ_d does not exceed 0.1 in the ultraviolet and as a rule is less than 0.001–0.01 in the infrared ($\lambda > 1$ μm). This is consistent with our assumption (iv).

The spatial distribution of grains (their number density) behaves as r^{-2} in the region of 'flying-away' whereas there is some deviation from this dependence in the region of intense grain growth because of the factor $[v = u(r)]^{-1}$ (equation 23).

Finally, we calculated the critical magnetic field strengths $B_{cr}(r)$ for each star. Fig. 3 represents typical behaviour of $B_{cr}(r)$ for the example of WR 106. At initial distances, where a grain starts to grow, a rise of B_{cr} up to some maximum may be seen. The maximum corresponds to the distance where the main grain parameters (size, velocity, electrical charge) become almost steady. At larger distances B_{cr} is well approximated by the inverse square law:

$$B_{cr}(r) \approx B_2 \left(\frac{r_*}{r} \right)^2, \tag{26}$$

where B_2 is some constant parameter with a typical range 1000–2000 G. Fig. 3 also shows the behaviour with distance of the possible magnetic fields in wind $B(r)$ calculated using equation (7) for various parameters B_1. Obviously, if $B_1 \geq 1$ G then the Lorentz force should suppress radial drift of a grain induced by the stellar radiation pressure. Grain growth through the implantation of carbon ions will therefore also be impossible because the grain will not reach the necessary suprathermal drift velocities. It is very probable that in this case no dust will form at all. In the other case when $B_1 \leq 0.01$ G, we may expect negligible influence of the magnetic field on the grain growth, which is described by our physical model. We therefore conclude that $B_1 \sim 0.1$ G corresponds to some critical case when the Lorentz force may be comparable with the radiation pressure force acting on the smallest grains. In this case grain growth may still be possible, but the model should be more complicated, e.g. the grain can drift in both radial and azimuthal directions. Note that at quite large distances, where the grain parameters reach their asymptotic values, B may be $> B_{cr}$. This, however, should not result in a drastic change of our model because the grains have already grown at such distances. The radial grain drift will slow down and probably transform into azimuthal drift, but the grain will continue to move radially with the wind velocity v at least. As our theoretical model is successful in explaining the infrared spectra of the WC stars around which the dust shells were proved to exist (Williams 1995), we may conclude that there should be no essential influence of the magnetic fields on the physical model of dust proposed by us, otherwise no dust will grow. We have found that the critical B_1s for all our stars do not differ essentially from each other and lie in the range 0.1–0.5 G. Putting $r_*/r_A \approx 1$ in equation (8) (it is strictly valid for $B \leq 10$ G), we obtain the constraint $B_s(v_{rot}/v) < 0.1$–0.5 G. As nothing is known about the rotation velocities of WC stars at present, we have to speculate. Assuming $v_{rot}/v \sim 0.1$ (typical for OB stars), we obtain $B_s < 1$–5 G, or if $v_{rot} \sim 0.01$ we obtain $B_s < 10$–50 G. We therefore see that, for our theoretical model to be valid, the surface magnetic fields should be quite moderate under the rotation velocities assumed. These estimates seem to be reasonable in the light of what we know about WC stars at present.

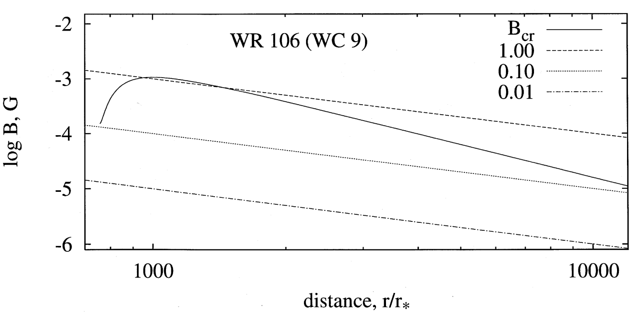

Figure 3. Critical and model magnetic fields in the wind of WR 106. Model fields are marked by values of the parameter B_1 in G (see text for more details).

4 CONCLUSIONS

(i) The observational infrared spectra of 17 WC stars can be quite satisfactorily explained in terms of the detailed physical model with amorphous carbon grains by taking into account the effects of the grain dynamics, temperature, electrical charging and growth–destruction.

(ii) The dust grains moving with suprathermal velocities up to 100–160 km s^{-1} may grow as a result of the implantation of impinging carbon ions. An opposite process, the grain destruction through kinetic sputtering by plasma species, restricts the final grain radii to within 100-200 Å.

(iii) The dust grains are mainly positively charged with electrostatic potential up to 1–1.2 V. Photoelectron emission and collisions with impinging plasma species are the main processes defining the equilibrium grain charge.

(iv) For the most of stars the fraction of condensed carbon does not exceed 1 per cent.

(v) The condensation nuclei production at the expected nucleation distances has been derived by comparing the theoretical spectra with observational ones.

(vi) A WC star dust shell consists of a region of intense grain growth followed by a region of grain 'flying-away' where the main grain parameters' radius, drift velocity and electrical potential, reach their asymptotic values.

(vii) Our physical model of dust around a WC star is valid provided that the strength of the surface magnetic field and the rotation velocity obey the constraint $B_s(v_{rot}/v) < 0.1$–0.5 G. If they do not, no dust will grow at all, but this contradicts the observations.

ACKNOWLEDGMENTS

It is a pleasure to thank Drs P. M. Williams, M. F. Bode, A. Evans, M. Cohen, W.-R. Hamann, S. V. Marchenko, T. Nugis and K. Annuk for helpful discussions. In addition, we are grateful to Dr P. M. Williams for providing us with his procedure for dereddening and converting the stellar photometric data into spectral fluxes and to Dr M. Cohen for making his paper on *IRAS* data for WC stars available to us before its publication. The helpful discussions with Dr S. V. Marchenko about the magnetic fields in WR stars are acknowledged with thanks. We are also grateful to the anonymous referee for valuable and constructive suggestions that helped to improve the paper. The model calculations have been performed with a Sun SPARCstation 5 bought from grant No. MEN/NSF-94-196 of the US–Polish Maria Skłodowska-Curie Joint Fund II.

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